

Table of Basic Integrals

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \quad (1)$$

$$\int \frac{1}{x} dx = \ln |x| \quad (2)$$

$$\int u dv = uv - \int v du \quad (3)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \quad (4)$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \quad (5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, \quad n \neq -1 \quad (6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \quad (7)$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \quad (8)$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (9)$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \quad (10)$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \quad (11)$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \quad (12)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (13)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \quad a \neq b \quad (14)$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \quad (15)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (16)$$

Integrals with Roots

$$\int \sqrt{x-a} \, dx = \frac{2}{3}(x-a)^{3/2} \quad (17)$$

$$\int \frac{1}{\sqrt{x \pm a}} \, dx = 2\sqrt{x \pm a} \quad (18)$$

$$\int \frac{1}{\sqrt{a-x}} \, dx = -2\sqrt{a-x} \quad (19)$$

$$\int x\sqrt{x-a} \, dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \quad (20)$$

$$\int \sqrt{ax+b} \, dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} \quad (21)$$

$$\int (ax+b)^{3/2} \, dx = \frac{2}{5a}(ax+b)^{5/2} \quad (22)$$

$$\int \frac{x}{\sqrt{x \pm a}} \, dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \quad (23)$$

$$\int \sqrt{\frac{x}{a-x}} \, dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \quad (24)$$

$$\int \sqrt{\frac{x}{a+x}} \, dx = \sqrt{x(a+x)} - a \ln [\sqrt{x} + \sqrt{x+a}] \quad (25)$$

$$\int x\sqrt{ax+b} \, dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \quad (26)$$

$$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \right] \quad (27)$$

$$\int \sqrt{x^3(ax+b)} dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \quad (28)$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \quad (29)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \quad (30)$$

$$\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2} \quad (31)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| \quad (32)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \quad (33)$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \quad (34)$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \quad (35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \quad (36)$$

$$\int \sqrt{ax^2 + bx + c} dx = \frac{b+2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left| 2ax+b+2\sqrt{a(ax^2+bx+c)} \right| \quad (37)$$

$$\begin{aligned} \int x\sqrt{ax^2 + bx + c} dx &= \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2 + bx + c} (-3b^2 + 2abx + 8a(c + ax^2)) \right. \\ &\quad \left. + 3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| \right) \end{aligned} \quad (38)$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \quad (39)$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \quad (40)$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} \quad (41)$$

Integrals with Logarithms

$$\int \ln ax \, dx = x \ln ax - x \quad (42)$$

$$\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \frac{x^2}{4} \quad (43)$$

$$\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{x^3}{9} \quad (44)$$

$$\int x^n \ln x \, dx = x^{n+1} \left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right), \quad n \neq -1 \quad (45)$$

$$\int \frac{\ln ax}{x} \, dx = \frac{1}{2} (\ln ax)^2 \quad (46)$$

$$\int \frac{\ln x}{x^2} \, dx = -\frac{1}{x} - \frac{\ln x}{x} \quad (47)$$

$$\int \ln(ax + b) \, dx = \left(x + \frac{b}{a} \right) \ln(ax + b) - x, \quad a \neq 0 \quad (48)$$

$$\int \ln(x^2 + a^2) \, dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \quad (49)$$

$$\int \ln(x^2 - a^2) \, dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x \quad (50)$$

$$\int \ln(ax^2 + bx + c) \, dx = \frac{1}{a} \sqrt{4ac - b^2} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left(\frac{b}{2a} + x \right) \ln(ax^2 + bx + c) \quad (51)$$

$$\int x \ln(ax + b) \, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2} \left(x^2 - \frac{b^2}{a^2} \right) \ln(ax + b) \quad (52)$$

$$\int x \ln(a^2 - b^2x^2) \, dx = -\frac{1}{2}x^2 + \frac{1}{2} \left(x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - b^2x^2) \quad (53)$$

$$\int (\ln x)^2 \, dx = 2x - 2x \ln x + x(\ln x)^2 \quad (54)$$

$$\int (\ln x)^3 \, dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x \quad (55)$$

$$\int x(\ln x)^2 \, dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x \quad (56)$$

$$\int x^2(\ln x)^2 \, dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3 \ln x \quad (57)$$

Integrals with Exponentials

$$\int e^{ax} \, dx = \frac{1}{a}e^{ax} \quad (58)$$

$$\int \sqrt{x}e^{ax} \, dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}(i\sqrt{ax}), \text{ where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (59)$$

$$\int xe^x \, dx = (x - 1)e^x \quad (60)$$

$$\int xe^{ax} \, dx = \left(\frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \quad (61)$$

$$\int x^2e^x \, dx = (x^2 - 2x + 2)e^x \quad (62)$$

$$\int x^2e^{ax} \, dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \quad (63)$$

$$\int x^3e^x \, dx = (x^3 - 3x^2 + 6x - 6)e^x \quad (64)$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad (65)$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \text{ where } \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt \quad (66)$$

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a}) \quad (67)$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a}) \quad (68)$$

$$\int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2} \quad (69)$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \quad (70)$$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a} \cos ax \quad (71)$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \quad (72)$$

$$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \quad (73)$$

$$\int \sin^n ax dx = -\frac{1}{a} \cos ax {}_2F_1 \left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax \right] \quad (74)$$

$$\int \cos ax dx = \frac{1}{a} \sin ax \quad (75)$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad (76)$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a} \quad (77)$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1 \left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax \right] \quad (78)$$

$$\int \cos x \sin x \, dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3 \quad (79)$$

$$\int \cos ax \sin bx \, dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \quad (80)$$

$$\int \sin^2 ax \cos bx \, dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \quad (81)$$

$$\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x \quad (82)$$

$$\int \cos^2 ax \sin bx \, dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \quad (83)$$

$$\int \cos^2 ax \sin ax \, dx = -\frac{1}{3a} \cos^3 ax \quad (84)$$

$$\int \sin^2 ax \cos^2 bxdx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \quad (85)$$

$$\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \quad (86)$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln \cos ax \quad (87)$$

$$\int \tan^2 ax \, dx = -x + \frac{1}{a} \tan ax \quad (88)$$

$$\int \tan^n ax \, dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \quad (89)$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \quad (90)$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right) \quad (91)$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax \quad (92)$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \quad (93)$$

$$\int \sec x \tan x \, dx = \sec x \quad (94)$$

$$\int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x \quad (95)$$

$$\int \sec^n x \tan x \, dx = \frac{1}{n} \sec^n x, n \neq 0 \quad (96)$$

$$\int \csc x \, dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C \quad (97)$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax \quad (98)$$

$$\int \csc^3 x \, dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x| \quad (99)$$

$$\int \csc^n x \cot x \, dx = -\frac{1}{n} \csc^n x, n \neq 0 \quad (100)$$

$$\int \sec x \csc x \, dx = \ln |\tan x| \quad (101)$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \quad (102)$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (103)$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \quad (104)$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (105)$$

$$\int x^n \cos x dx = -\frac{1}{2}(i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)] \quad (106)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)] \quad (107)$$

$$\int x \sin x \, dx = -x \cos x + \sin x \quad (108)$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (109)$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \quad (110)$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (111)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (112)$$

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \quad (113)$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \quad (114)$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \quad (115)$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \quad (116)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (117)$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (118)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (119)$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (120)$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (121)$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (122)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax \quad (123)$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (124)$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \quad (125)$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (126)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (127)$$

$$\int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)^2} {}_2F_1 \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (128)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (129)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (130)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (131)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (132)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (133)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (134)$$